Jour. Ind. Soc. Ag. Statistics
Vol. XL, No. 2 (1988), pp. 150-153

A NOTE ON EQUIREPLICATE BALANCED BLOCK DESIGNS FROM BIB DESIGNS

KISHORE SINHA
Birsa Agricultural University, Ranchi-834006
(Received : September, 1987)

SUMMARY

A method of construction of equireplicate balanced block designs with unequal block sizes from balanced incomplete block (BIB) designs, together with a table of new designs in the range $r \leqslant 30$ have been given.

Keywords : Equireplicate balanced block design; BIB design; t-design.

Introduction

A block design is called variance x-balanced if all elementary treatment contrasts are estimated with same variance. A connected block design is variance-balanced iff,

$$C = \theta (I - v^{-1}J), \qquad \text{where } \theta \text{ is a constant.}$$

A connected block design is efficiency-balanced iff,

$$NK^{-1}N' = \mu R + \{(1 - \mu)/n\}\, rr' \qquad \text{where } r' = (r_1, r_2, \ldots, r_v),$$

$R = \text{diag}\,(r_1, r_2, \ldots, r_v)$, $K = \text{diag}\,(k_1, k_2, \ldots, k_b)$, n is number of experimental units, μ is constant and the efficiency factor of the design is $1 - \mu$.

The balanced block designs considered here are equireplicate, hence these are variance-balanced as well as efficiency-balanced. The construction and tabulation of equireplicate balanced block designs have been studied by Gupta and Jones [2], Agarwal and Kumar [1], Jones, Sinha and Kageyama [3], Sinha and Jones [7], Sinha [5, 6, 8]. The designs have been listed in the range $r \leqslant 30$.

Here, a method of construction of equireplicate balanced block designs with unequal block sizes from balanced incomplete block (BIB) designs and in general from t-designs, together with a table of new designs in the range $r \leqslant 30$ have been given.

2. The Method

THEOREM 1. *The existence of a BIBD $(v', b', r', k', \lambda')$ implies the existence of equireplicate balanced block designs with parameters:*

$v = v' - p$, $r = r'k' - p\lambda'$, $p = 1, 2$, $k_i = k' - i + 1$, $i = 2, p + 1$
$b_1 = k' \{b' - pr' + (p - 1)^2 \lambda'\}$, $b_2 = p \{r' - (p - 1) \lambda'\} (k' - 1)$
$b_3 = (p - 1) \lambda' (k' - 2)$, ω (*the common sum of weighted concurrences*) $= \lambda'$. (2.1)

Proof. Without loss of generality, the last p treatments are deleted from the BIBD. Then the set of blocks of size $(k' - i + 1)$, $i = 1, 2, p + 1$, are repeated $(k' - i + 1)$ times to obtain an equireplicate balanced block design with parameters (2.1).

When $p = 1$, we obtain a set of r' blocks of size $(k' - 1)$ and replication λ'; and, another set of $b' - r'$ blocks of size k' with replication $r' - \lambda'$.

When $p = 2$, in the blocks of size $k' - 2$, we have $r_1 = 1$, $r_2 = 0$; in the blocks of size $k' - 1$, we have $r_1 = 2 (\lambda' - 1)$, $r_2 = 2\lambda'$; and in the blocks of size k', we have $r_1 = r' - 2 \lambda' + 1$, $r_2 = r' - 2 \lambda'$. Since over all the sets of blocks, obtained by deleting the last two treatments of the BIBD, the design is pairwise-balanced design with $\lambda = \lambda'$, and blocks of sizes $k' - i + 1$ are repeated $k' - i + 1$ times, we get the common sum of weighted concurrences as $\omega = \lambda'$. In general, p treatments $(1 \leqslant p \leqslant t)$ may be deleted from a t-design, to obtain equireplicate balanced block designs, in a manner analogous to the above theorem.

EXAMPLE. Let us consider a BIB design no. $R15$: $v = 8$, $b = 14$, $r = 7$, $k = 4$, $\lambda = 3$;

(0125) (1236) (0234) (1345) (2456) (0356) (0146)

(1247) (2357) (3467) (0457) (1567) (0267) (0137),

which is also a 3-design with $\lambda_3 = 1$.

Now by deleting $p = 1, 2, 3$ treatments, we get, respectively, equireplicate balanced designs :

(i) $v = 7, r = 25. k_i = 4, 3, b_1 = 28, b_2 = 21, \omega = 3, 100E = 84.00,$

(ii) $v = 6, r = 22, k_i = 4, 3, 2, b_1 = 12, b_2 = 24, b_3 = 6, \omega = 3, 100E = 81.82,$

(iii) $v = 5, r = 19, k_i = 4, 3, 2, 1, b_1 = 4, b_2 = 18, b_3 = 12, b_4 = 1, \omega = 3, 100E = 78.95.$

TABLE 1—NEW EQUIREPLICATE BALANCED BLOCK DESIGNS

Sl. No.	v	r	k_i. $i = 1, 2, 3$	b_1	b_2	b_3	ω	100E	Source (No. of treatments deleted)
1	5	7	$3 - i + 1$	6	8	1	1	71.43	R10 (2)
2	5	12	$4 - i + 1$	4	12	4	2	83.33	R11 (2)
3	5	13	3, 2	15	10	—	2	76 92	R7 (1)
4	6	22	$4 - i + 1$	12	24	6	3	81.82	R15 (2)
5	7	25	4, 3	28	21	—	3	84.00	R15 (1)
6	7	10	$3 - i + 1$	15	12	1	1	70.00	R17 (2)
7	7	26	$4 - i + 1$	20	30	6	3	80.77	R19 (2)
8	8	20	$4 - i + 1$	20	24	4	2	80.00	R23 (2)
9	8	23	$3 - i + 1$	42	28	2	2	69.57	R25 (2)
10	8	29	4, 3	40	24	—	3	82.76	R19 (1)
11	9	21	$5 - i + 1$	15	24	6	2	85.71	R29 (2)
12	9	25	3, 2	63	18	—	2	72.00	R25 (1)
13	9	30	$6 - i + 1$	12	30	12	3	90.00	R30 (2)
14	10	29	$3 - i + 1$	72	36	2	2	68.97	R34 (2)
15	11	14	$4 - i + 1$	24	18	2	1	78.57	R37 (2)
16	11	16	$3 - i + 1$	45	20	1	1	68.75	R38 (2)
17	13	19	$3 - i + 1$	66	24	1	1	68.42	R42 (2)
18	14	18	$4 - i + 1$	44	24	2	1	77.78	R46 (2)
19	17	25	$3 - i + 1$	120	32	1	1	68.00	R54 (2)
20	18	26	3, 2	144	18	—	1	69.23	R54 (1)
21	19	23	$5 - i + 1$	60	32	3	1	82.61	R58 (2)
22	19	28	$3 - i + 1$	153	36	1	1	67.86	R59 (2)
23	20	29	3, 2	180	20	—	1	68.96	R59 (1)
24	23	28	$5 - i + 1$	95	40	3	1	82.14	R65 (2)
25	23	30	$4 - i + 1$	140	42	2	1	76.67	R66 (2)
26	24	29	5, 4	120	24	—	1	82.76	R65 (1)

The R numbers are BIBDs from Raghavarao [4]. The efficiency factor of an equireplicate balanced block design is given by, $E = v\omega/r$.

ACKNOWLEDGEMENT

The author is highly thankful to the referee for useful suggestions.

REFERENCES

[1] Agarwal, G. G. and Kumar, S. (1984) : On a class of variance balanced designs associated with GD designs, *Calcutta Statistical Association Bull.* 33 : 187-90.

[2] Gupta, S. C. and Jones, B. (1983) : Equireplicate balanced block designs with unequal block sizes, *Biometrika* 70(2) : 443-40.

[3] Jones, B., Sinha, K. and Kageyama, S. (1987) : Further equireplicate variance balanced designs with unequal block sizes, *Utilitas Mathematica* 32 : 5-10.

[4] Raghavarao, D. (1971) : *Constructions and Combinatorial Problems in Design of Experiments*, John Wiley and Sons, New York.

[5] Sinha, K. (1987) : Generalized partially balanced incomplete block designs, *Discrete Mathematics* 67 : 31-58.

[6] Sinha, K. (1987) : Some new equireplicate balanced block designs, submitted to *Statistics and Probability Letters.*

[7] Sinha, K. and Jones, B. (1987) : Further equireplicate balanced block designs with unequal block sizes, *Statistics and Probability Letters* 6 : 229-230.

[8] Sinha, K. (1988) : Further generalization of partially balanced incomplete block designs. Submitted for publications.